

Assignment 4: Algorithms using Divide and Conquer

Competitive Programming

Question 1

Aim: You are given an array of k linked-lists lists, each linked-list is sorted in ascending order. Using divide and conquer, Merge all the linked-lists into one sorted linked-list and return it.

Algorithm

```
Merge K lists (L [0..k-1])
if k = 0 return null
while size(L) > 1
    temp ← empty list
    for i ← 0 to size(L) - 1 step 2
        list 1 ← L[i]
        if i + 1 < size(L)
            list 2 ← L[i + 1]
        else
            list 2 ← null
        merged ← Merge Two lists (list 1, list 2)
        Add merged to temp
    L ← temp
return L[0]

Merge Two lists (list 1, list 2)
dum ← new Node()
current ← dum
while list 1 is not null and list 2 is not null
    if list 1.val ≤ list 2.val
        current.next ← list 1
        list 1 ← list 1.next
    else
        current.next ← list 2
        list 2 ← list 2.next
    current ← current.next
if list 1 is not null current.next ← list 1
else current.next ← list 2
return dum.next
```

Code

```
class Solution:
    def mergeKLists(self, lists: List[Optional[ListNode]]) -> Optional[ListNode]:
        if not lists or len(lists) == 0:
            return None

        while len(lists) > 1:
            tmp = []
            for i in range(0, len(lists), 2):
                list_1 = lists[i]
                list_2 = lists[i+1] if i + 1 < len(lists) else None
                tmp.append(self.merge_lists(list_1, list_2))
            lists = tmp

        return lists[0]

    def merge_lists(self, list_1, list_2):
        node = ListNode()
        ans = node

        while list_1 and list_2:
            if list_1.val < list_2.val:
                node.next = list_1
                list_1 = list_1.next
            else:
                node.next = list_2
                list_2 = list_2.next
            node = node.next

        if list_1:
            node.next = list_1
        else:
            node.next = list_2

        return ans.next
```

Time complexity

Recurrence relation:

$$T(k) = 2T(k/2) + f(k)$$

Applying Master's Theorem

$A = 2, b = 2,$

$f(k) = \Theta(n)$

Comparing a with b^d ,

$$a = 2$$

$$b^d = 2^0 = 1$$

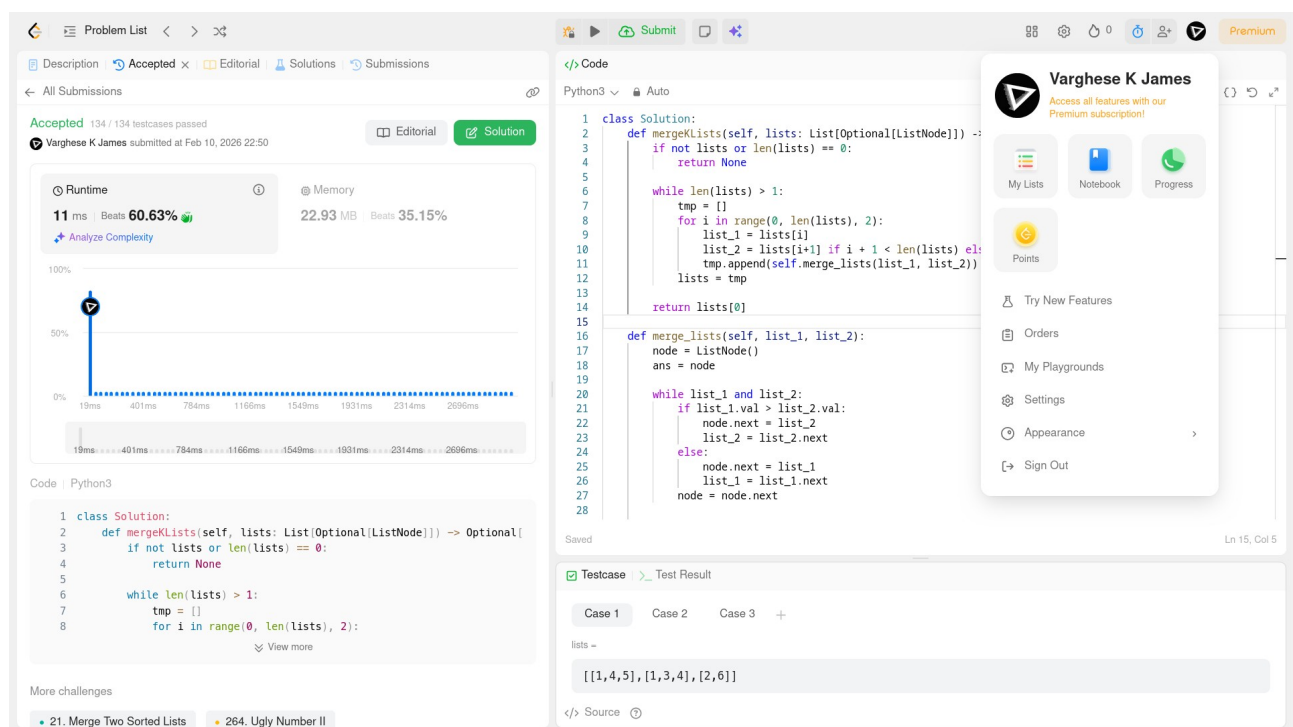
Since $a > b^d$, standard Master Theorem suggests

$$\Theta(k^{\log_b a}) = \Theta(k^1).$$

However, because the work at each level is actually n , there are $\log(k)$ levels.

$$T(n, k) = \Theta(n \log k)$$

Screenshot of Output

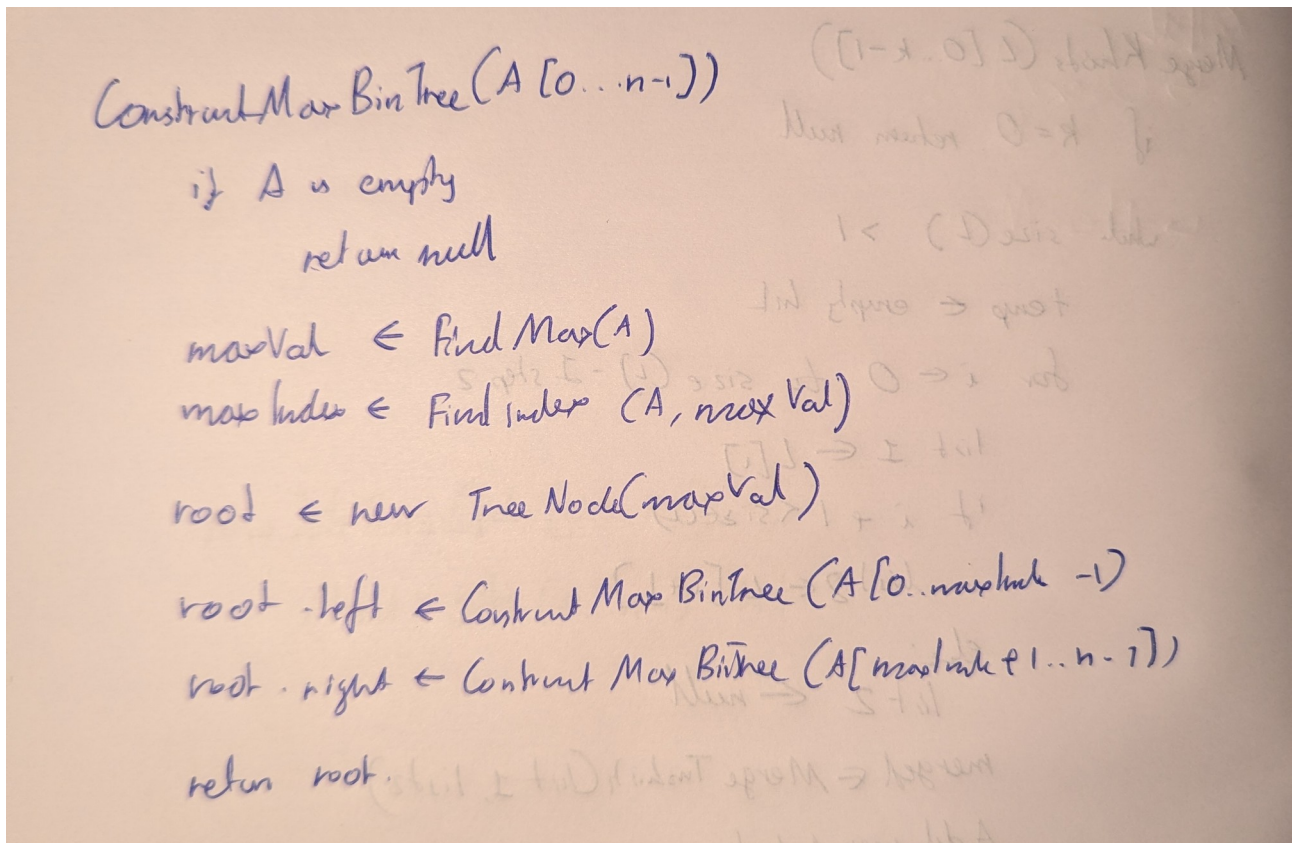


Question 2

Aim: You are given an integer array `nums` with no duplicates. A maximum binary tree can be built recursively from `nums` using the following algorithm:

- Create a root node whose value is the maximum value in `nums`.
- Recursively build the left subtree on the subarray prefix to the left of the maximum value.
- Recursively build the right subtree on the subarray suffix to the right of the maximum value.
- Return the maximum binary tree built from `nums`.

Algorithm



Code

```
class Solution(object):  
    def constructMaximumBinaryTree(self, nums):  
        if not nums:  
            return
```

```
element = max(nums)
index = nums.index(element)

node = TreeNode(element)

node.left = self.constructMaximumBinaryTree(nums[0:index])
node.right = self.constructMaximumBinaryTree(nums[index + 1:])

return node
```

Time complexity

Recurrence relation:

$$T(n) = T(i) + T(n - i - 1) + \Theta(n)$$

Applying Master's Theorem

$A = 2, b = 2,$

$$f(n) = n^1 \implies d = 1$$

Since $a = b^d$ ($2 = 2^1$), this is case 2 of Master's Theorem,

$$T(n) \in \Theta(n^d \log n) = \Theta(n \log n)$$

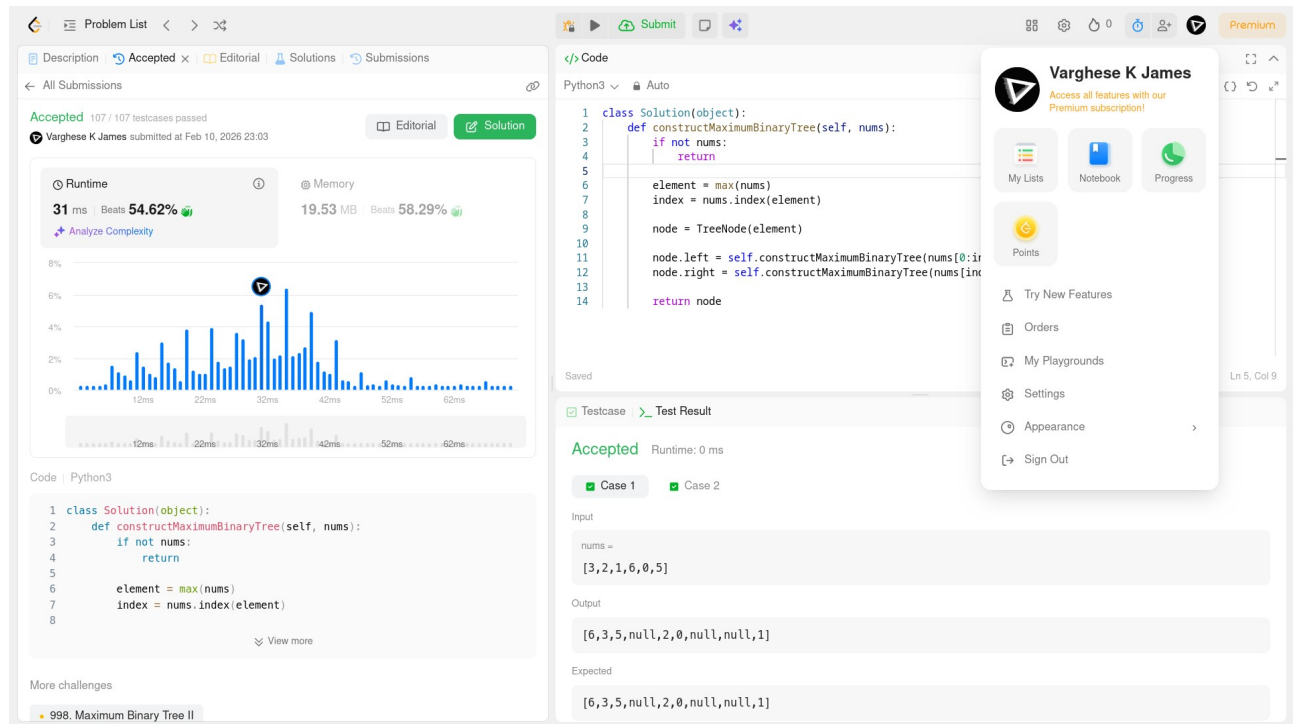
In the worst case,

$$T(n) = T(n - 1) + \Theta(n)$$

This is recursive summation and results in,

$$T(n) \in \Theta(n^2)$$

Screenshot of Output



Question 3

Aim: A city's skyline is the outer contour of the silhouette formed by all the buildings in that city when viewed from a distance. Given the locations and heights of all the buildings, return the skyline formed by these buildings collectively.

Algorithm

```
Skyline(B[0..n-1])
    n ← length(B)
    if n = 0 return []
    if n = 1
        [L, R, H] ← B[0]
        return [[L, H], [R, 0]]
    mid ← ⌊n/2⌋
    leftSkyline ← skyline(B[0..mid])
    rightSkyline ← skyline(B[mid..n-1])
    return MergeSkyline(leftSkyline, rightSkyline)

MergeSkyline(left, right)
    h1 ← 0, h2 ← 0, i ← 0, j ← 0
    result ← empty list
    while i < length(left) and j < length(right)
        if left[i].x < right[j].x
            x ← left[i].x
            h1 ← left[i].h
            i ← i + 1
        else if left[i].x > right[j].x
            x ← right[j].x
            h2 ← right[j].h
            j ← j + 1
        else
            x ← left[i].x
            h1 ← left[i].h
            h2 ← right[j].h
            i ← i + 1
            j ← j + 1
    maxHeight ← max(h1, h2)
    if result is empty or result.last.h ≠ maxHeight
        Append [x, maxHeight] to result
```

Code

```
class Solution:
    def getSkyline(self, buildings: List[List[int]]) -> List[List[int]]:
        n = len(buildings)
        if n == 0: return []
        if n == 1:
            L, R, H = buildings[0]
            return [[L, H], [R, 0]]

        mid = n // 2
        left = self.getSkyline(buildings[:mid])
        right = self.getSkyline(buildings[mid:])
        return self.merge(left, right)

    def merge(self, left: List[List[int]], right: List[List[int]]) -> List[List[int]]:
        h1, h2 = 0, 0
        i, j = 0, 0
        res = []
        n_l, n_r = len(left), len(right)

        while i < n_l and j < n_r:
            if left[i][0] < right[j][0]:
                x, h1 = left[i]
                i += 1
            elif left[i][0] > right[j][0]:
                x, h2 = right[j]
                j += 1
            else:
                x, h1 = left[i]
                h2 = right[j][1]
                i += 1
                j += 1

            h = max(h1, h2)
            if not res or res[-1][1] != h:
                res.append([x, h])

        res.extend(left[i:] or right[j:])
        return res
```

Time complexity

Recurrence relation:

$$T(n) = 2T(n/2) + \Theta(n)$$

Applying Master's Theorem:

$a = 2, b = 2,$

$$f(n) = \Theta(n^1) \implies d = 1$$

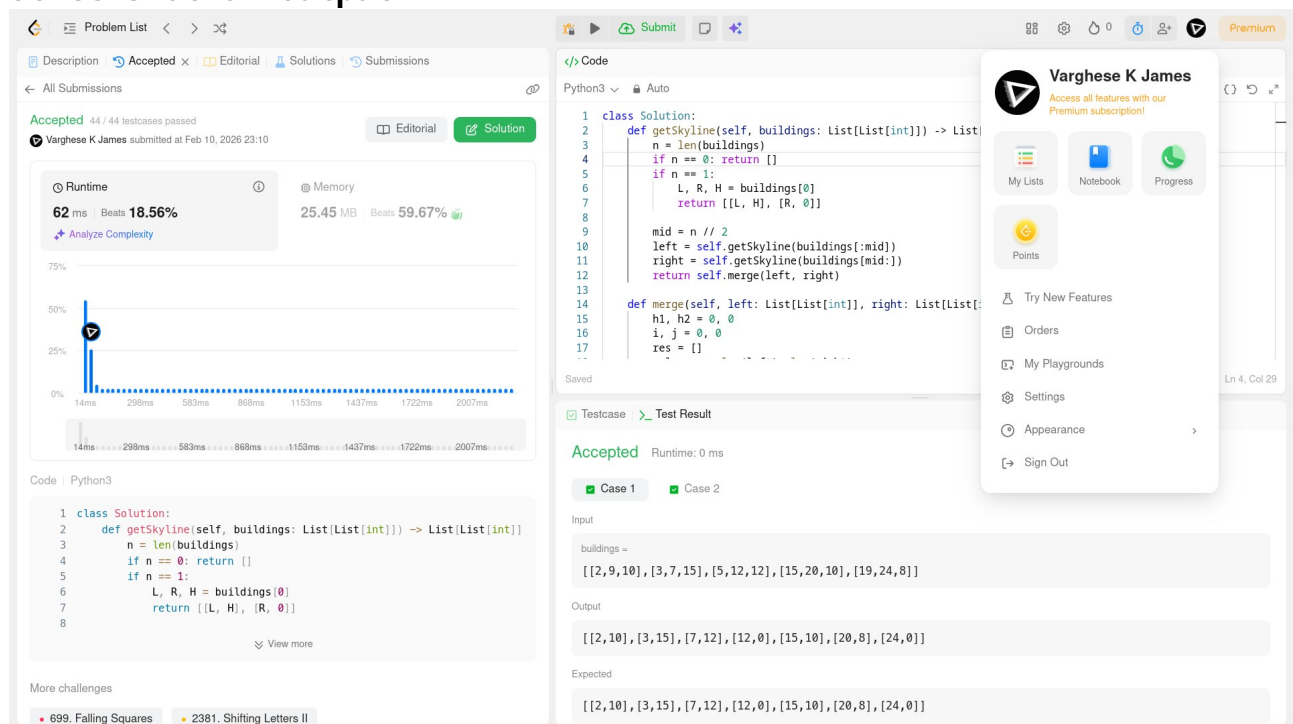
Since, $a = b^d$ ($2 = 2$), it is case 2 of Master's Theorem

$$T(n) \in \Theta(n^d \log n)$$

Substituting $d = 1,$

$$T(n) \in \Theta(n \log n)$$

Screenshot of Output



Learning Outcomes

- Learned about using divide and conquer algorithms to solve questions.
- Learned about optimisations done to solve competitive programming questions.