

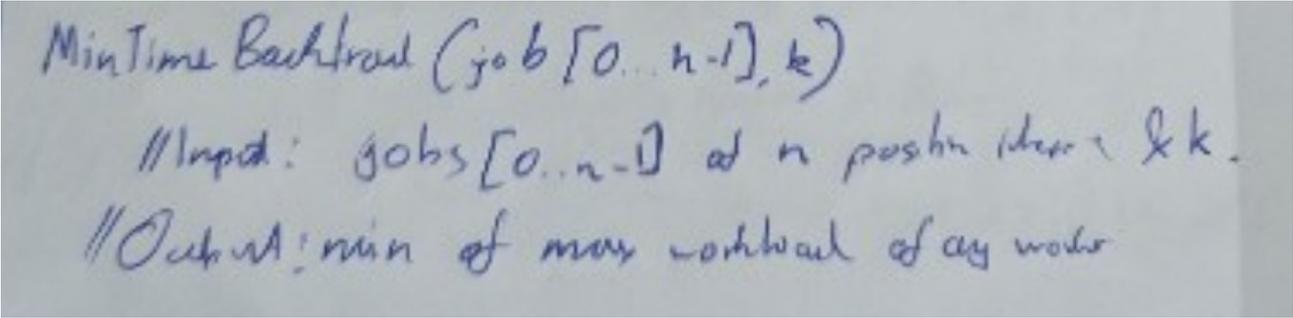
Assignment 3: Algorithms Using Backtracking

Competitive Programming

Question 1

Aim: You are given an integer array of jobs, where jobs[i] is the amount of time it takes to complete the ith job. There are k workers that you can assign jobs to. Each job should be assigned to exactly one worker. The working time of a worker is the sum of the time it takes to complete all jobs assigned to them. Your goal is to devise an optimal assignment such that the maximum working time of any worker is minimized. Return the minimum possible maximum working time of any assignment.

Algorithm



MinTime Backtrack (job [0..n-1], k)
// Input: jobs [0..n-1] of n positions & k.
// Output: min of max workload of any worker

```
workers[0..k-1] ← [0, 0, ..., 0]
res ← ∞

Backtrack(0, res)
return res

Backtrack(cur-job, res)
  if cur-job = n then
    res ← min(res, max(workers))
    return
  seen ← ∅
  for i ← 0 to k-1 do
    if workers[i] ∈ seen then
      continue
    if worker[i] + job[cur-job] ≥ res then
      continue
    add worker[i] to seen
    workers[i] ← workers[i] + job[cur-job]
    Backtrack(cur-job + 1, res)
  workers[i] ← worker[i] - job[cur-job]
```

Code

```
class Solution:
    def minimumTimeRequired(self, jobs: List[int], k: int) -> int:
        workers = [0]*k

        self.res = sys.maxsize

        def backtrack(cur_job):
            if cur_job == len(jobs):
                self.res = min(self.res, max(workers))
                return

            seen = set()
            for i in range(k):
                if workers[i] in seen: continue
                if workers[i] + jobs[cur_job] >= self.res: continue
                seen.add(workers[i])
                workers[i] += jobs[cur_job]
                backtrack(cur_job+1)
                workers[i] -= jobs[cur_job]

        backtrack(0)
        return self.res
```

Time complexity

Recurrence relation is:

$$T(j) = k \cdot T(j - 1) + c$$

By using substitution method,

$$T(n) = k \cdot T(n - 1) + c$$

$$T(n) = k[k \cdot T(n - 2) + c] + c = k^2 T(n - 2) + kc + c$$

$$T(n) = k^2[k \cdot T(n - 3) + c] + kc + c = k^3 T(n - 3) + k^2 c + kc + c$$

$$T(n) = k^n T(0) + c \sum_{i=0}^{n-1} k^i$$

$$T(n) = k^n \cdot k + c \left(\frac{k^n - 1}{k - 1} \right)$$

Using asymptotic notation it becomes k^n , hence:

$$O(k^n)$$

Screenshot of Output

The screenshot displays a LeetCode submission for problem 1986, "Minimum Number of Work Sessions to Finish the Tasks". The submission is in Python3 and has a runtime of 2441 ms, which beats 42.22% of other submissions. The memory usage is 19.49 MB, beating 46.87%. The code uses a backtracking approach to find the minimum number of work sessions required to finish the tasks.

```
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    def minimumTimeRequired(self, jobs: List[int], k: int) -> int:
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        self.res = sys.maxsize

        def backtrack(cur_job):
            if cur_job == len(jobs):
                self.res = min(self.res, max(workers))
                return

            seen = set()
            for i in range(k):
                if workers[i] in seen: continue
                if workers[i] + jobs[cur_job] >= self.res: continue
                seen.add(workers[i])
                workers[i] += jobs[cur_job]
                backtrack(cur_job+1)
                workers[i] -= jobs[cur_job]

        backtrack(0)
        return self.res
```

Learning Outcomes

- Learned about using backtracking algorithms to solve questions involving permutations and combination
- Learned about optimisations done to solve competitive programming questions.