

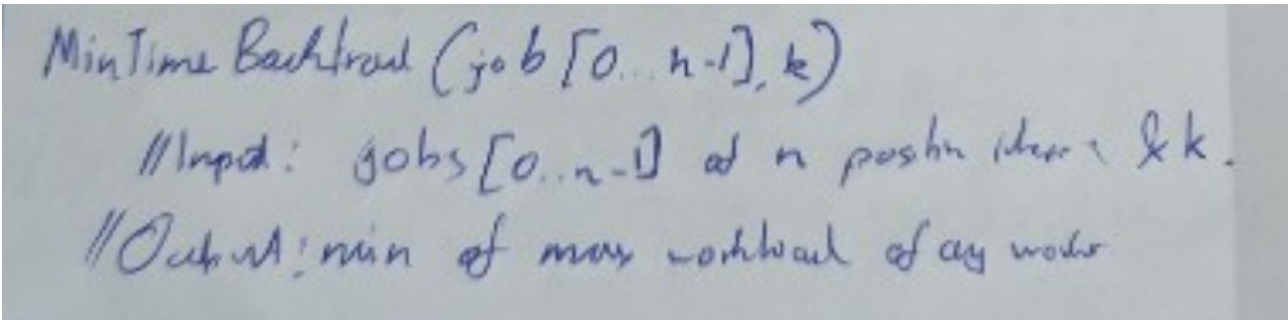
## Assignment 3: Algorithms Using Backtracking

### Competitive Programming

#### Question 1

**Aim:** You are given an integer array of jobs, where  $jobs[i]$  is the amount of time it takes to complete the  $i$ th job. There are  $k$  workers that you can assign jobs to. Each job should be assigned to exactly one worker. The working time of a worker is the sum of the time it takes to complete all jobs assigned to them. Your goal is to devise an optimal assignment such that the maximum working time of any worker is minimized. Return the minimum possible maximum working time of any assignment.

#### Algorithm



MinTime Backtrack ( $jobs[0..n-1], k$ )  
// Input:  $jobs[0..n-1]$  of  $n$  positions and  $k$ .  
// Output: min of max workload of any worker

$workers[0..k-1] \leftarrow [0, 0, \dots, 0]$

$res \leftarrow \infty$

Backtrack(0, res)

return res

Backtrack(cur-job, res)

if cur-job = n then

$res \leftarrow \min(res, \max(workers))$

return

seen  $\leftarrow \emptyset$

for  $i \leftarrow 0$  to  $k-1$  do

if  $workers[i] \in \text{seen}$  then

continue

if  $worker[i] + job[cur-job] \geq res$  then

continue

add  $worker[i]$  to seen

$workers[i] \leftarrow workers[i] + job[cur-job]$

Backtrack(cur-job + 1, res)

$workers[i] \leftarrow worker[i] - jobs[cur-job]$

## Code

```
class Solution:
    def minimumTimeRequired(self, jobs: List[int], k: int) -> int:
        workers = [0]*k

        self.res = sys.maxsize

        def backtrack(cur_job):
            if cur_job == len(jobs):
                self.res = min(self.res, max(workers))
                return

            seen = set()
            for i in range(k):
                if workers[i] in seen: continue
                if workers[i] + jobs[cur_job] >= self.res: continue
                seen.add(workers[i])
                workers[i] += jobs[cur_job]
                backtrack(cur_job+1)
                workers[i] -= jobs[cur_job]

        backtrack(0)
        return self.res
```

## Time complexity

Recurrence relation is:

$$T(j) = k \cdot T(j - 1) + c$$

By using substitution method,

$$T(n) = k \cdot T(n - 1) + c$$

$$T(n) = k[k \cdot T(n - 2) + c] + c = k^2 T(n - 2) + kc + c$$

$$T(n) = k^2[k \cdot T(n - 3) + c] + kc + c = k^3 T(n - 3) + k^2 c + kc + c$$

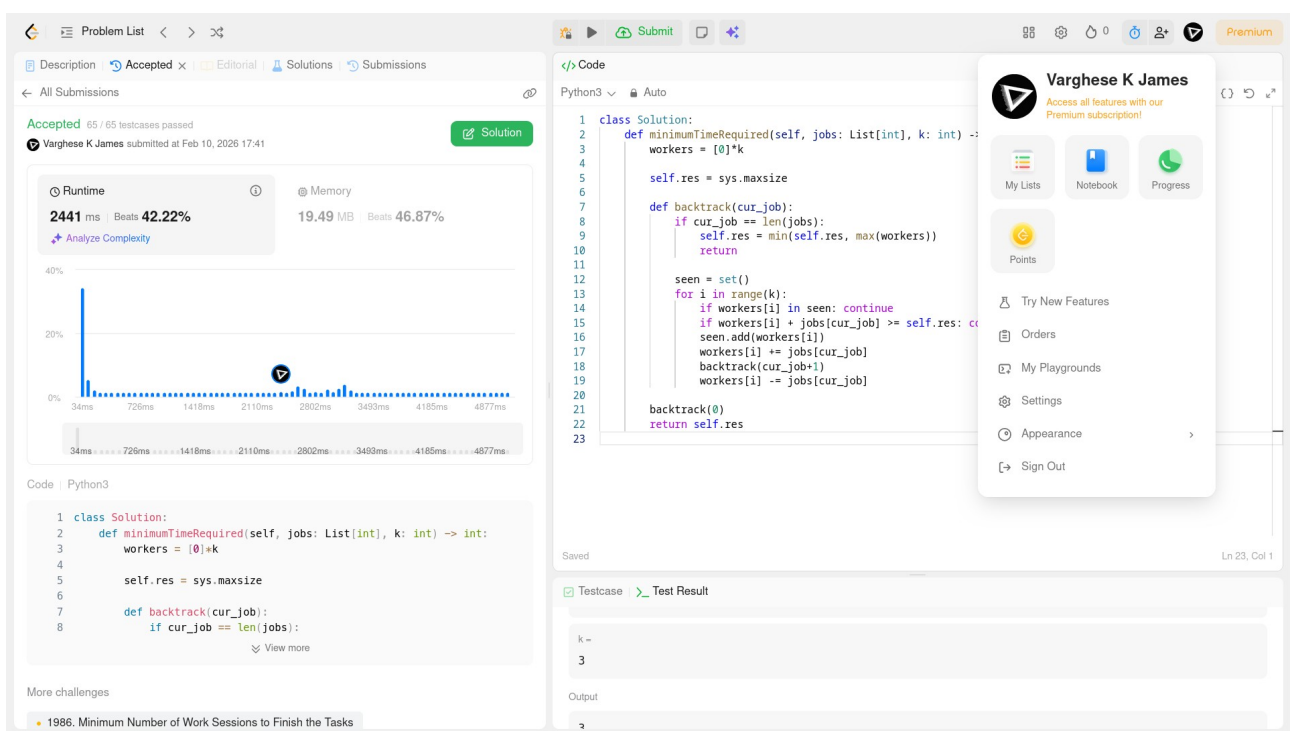
$$T(n) = k^n T(0) + c \sum_{i=0}^{n-1} k^i$$

$$T(n) = k^n \cdot k + c \left( \frac{k^n - 1}{k - 1} \right)$$

Using asymptotic notation it becomes  $k^n$ , hence:

$$O(k^n)$$

## Screenshot of Output



## Learning Outcomes

- Learned about using backtracking algorithms to solve questions involving permutations and combination
- Learned about optimisations done to solve competitive programming questions.